ENTROPY SOLUTION OF NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS WITH DIFFUSE MEASURE DATA

Safimba Soma and Mohamed Bance

Abstract

Given a parabolic cylinder $Q_T = \Omega \times (0, T)$, where Ω is a bounded domain of $\mathbb{R}^N$, we consider the nonlinear integro-differential parabolic problems with Dirichlet boundary values of the type

$$\partial_t (k * (b(v) - b(v_0))) - \operatorname{div}(a(x, Dv) + F(v)) = \mu \quad \text{in} \quad Q_T,$$

where b is a non-decreasing $\mathcal{C}^0$-function, kernel k belongs to the large class of $\mathcal{PC}$ kernels and μ is a diffuse measure. We prove the existence of an entropy solution for this class of nonlinear parabolic equations.

References

[1] K. Ammar and P. Wittbold, Existence of renormalized solutions of degenerate elliptic-parabolic problems, Proc. Roy. Edinburgh Sect. A 133 (2003), 477-496.

[2] L. Boccardo, Some nonlinear Dirichlet problems in L^1-involving lower order terms in divergence form, In Progress in Elliptic and Parabolic Partial Differential Equations (1996), pp. 43-57.

[3] Ph. Bénilan and P. Wittbold, On mild and weak solutions of elliptic-parabolic problems, Adv. Diff. Equ. 1(6) (1996), 1053-1073.

[4] M. Caputo, Diffusion of fluids in porous media with memory, Geothermics 28(1) (1999), 113-130.

[5] Ph. Clement and E. Mitidieri, Qualitative properties of solutions of volterra equations in Banach spaces, Israel J. Math. 64 (1988), 1-24.

[6]   Ph. Clément, On the abstract volterra equations in Banach spaces with completely kernels, Infinite-dimensional systems, Lecture Notes Math., 1984, pp 32-40.

[7]   S.-O. Londen, G. Gripenberg and O. J. Staffans, Volterra integral and functional equations, Encyclopedia of Mathematics and its Applications, Cambridge University Press, 1990.

[8]   L. Boccardo, T. Gallouët and L. Orsina, Existence and uniqueness of entropy solutions for nonlinear elliptic equations with measure data, Ann. Inst. Poincaré Anal. Non Linéaire 13 (1996), 539-551.

[9]   G. Gripenberg, Volterra integro-differential equations with accretive nonlinearity, J. Differ. Equat. 60 (1985), 57-79.

[10] V. G. Jakubowski, Nonlinear elliptic-parabolic integro-differential equations with L^1- data: existence, uniqueness, asymptotics, PhD. Thesis, University Duisburg-Essen, 2011.

[11] V. G. Jakubowski and P. Wittbold, On a nonlinear elliptic-parabolic integro-differential equation with L^1- data, J. Differ. Equat. 197 (2004), 427-445.

[12] R. Landes, On the existence of weak solutions for quasilinear parabolic initial-boundary value problems, Proc. R. Edinb. A 89 (1981), 217-237.

[13] R. Metzler and J. Klafter, The random walk's guide to anomalous diffusion: a fractional dynamics approach, Phys. Rep. 339 (2000), 1-77.

[14] T. Gallouet, R. Gariepy, M. Pierre, J. L. Varquez, Ph. Benilan and L. Boccardo, An L^1- theory of existence and uniqueness of solutions of nonlinear elliptic equations, Ann. Scuola. Norm. Sup. Pisa 22(2) (1991), 241-273.

[15] A. Prignet, Remarks on the existence and uniqueness of solutions of elliptic problems with right-hand side measure, Rend. Mat. Appl. 22(3) (1995), 321-337.

[16] J. Droniou, A. Porretta and A. Prignet, Parabolic capacity and soft measures for nonlinear equations, Potential Anal. 19 (2003), 99-161.

[17] N. Sapountzoglou, Entropy solutions to doubly nonlinear integro-differential equations, Nonlinear Anal. 192 (2020), 31.

[18] M. Scholtes, Entropy solutions to nonlinear integro-differential equations, PhD. Thesis, Universität Duisburg-Essen, 2016.

[19] Martin Scholtes and Petra Wittbold, Existence of entropy solutions to a doubly nonlinear integro-differential equation, Differ. Integral Equ. 31(5-6) (2018), 465-496.

[20] J. Serrin, Pathological solutions of elliptic differential equations, Annali della Scuola Normale Superiore di Pisa-Scienze Fisiche e Matematiche 18 (1964), 385-387.

[21]   S. Soma and M. Bance, On existence of entropy solution for a doubly nonlinear differential equation with L^1- data, J. Appl. Math. Phys. 11(12) (2023), 4092-4127.

[22]   V. Vergara and R. Zacher, A priori bounds for degenerate and singular evolutionary partial integro-differential equations, Nonlinear Anal. 73 (2010), 3572-3585.

[23]   V. Vergara and R. Zacher, Optimal decay estimates for time-fractional and other nonlocal subdiffusion equations via energy methods, SIAM J. Math. Anal. 47 (2015), 210-239.

[24]   R. Zacher, Weak solutions of abstract evolutionary integro-differential equations in Hilbert spaces, Funccial. Ekvac. 52 (2009), 1-18.

[25]   A. Zimmermann, Renormalized solutions for nonlinear partial differential equations with variable exponents and L^1- data, PhD. Thesis, University of Berlin, 2010.