## OUTPUT TRACKING CONTROL OF A DIFFUSION PROCESS WITH BOUNDARY INPUTS

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## Abstract

In this paper, a finite-dimensional output tracking control problem is considered for the one-dimensional diffusion system with two Neumann boundary inputs and one boundary output. The output tracking control method proposed here is a finite-dimensional one such that the boundary output approaches any fixed reference signal and the closed-loop stability is assured. For that purpose, we first find an equilibrium point of system for a given reference signal and then set a feedforward control law for one boundary input. By defining the error variable as the difference between the state variable and the equilibrium point, the problem is equivalent to the one of stabilizing the error system by using the other boundary input. It is shown that the error system is stabilized by a feedback control law containing residual mode filter (RMF). Combining the feedback control law with the feedforward control law mentioned above, the output tracking can be achieved for the original system. A numerical simulation result is given to demonstrate our design method.

**Keywords and phrases:** one-dimensional diffusion equation, Neumann boundary control, boundary observation, output tracking.

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