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# ON TERNARY QUADRATIC DIOPHANTINE EQUATION $2(x^2 + y^2) - 3xy = 43z^2$

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**Abstract**

The ternary quadratic equation $2(x^2 + y^2) - 3xy = 43z^2$ representing cone is analyzed by its nonzero distinct integer points on it. Employing the integer solutions, a few relations between the solutions and special polygonal numbers are presented.

## 1. Introduction

The ternary quadratic Diophantine equation offers an unlimited field for research because of their variety [1-2]. In particular, one may refer [3-23] for finding points on some specific three dimensional surfaces. This communication concerns with yet another ternary quadratic equation $2(x^2 + y^2) - 3xy = 43z^2$ representing cone for determining its infinitely many integer solutions. Employing integral solutions on the cone, a few interesting relations among the special polygonal and pyramidal numbers are given.

**Notations.**

$$t_{m,\,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right).$$

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$$SO_n = n(2n^2 - 1).$$

$$P_n^5 = \frac{n^2(n+1)}{2}.$$

$$\Pr_n = n(n+1).$$

$$OH_n = \frac{1}{3}[n(2n^2 + 1)].$$

## 2. Method of Analysis

Consider the equation

$$2(x^2 + y^2) - 3xy = 43z^2. \tag{1}$$

The substitution of linear transformations

$$x = u + v; \quad y = u - v \quad (u \neq v \neq 0) \tag{2}$$

in (1) leads to

$$u^2 + 7v^2 = 43z^2. \tag{3}$$

The above equation is solved through different methods and using (2), different patterns of integer solutions to (1) are obtained.

**Pattern 1**

Write 43 as

$$43 = (6 + i\sqrt{7})(6 - i\sqrt{7}). \tag{4}$$

Assume

$$Z = a^2 + 7b^2, \tag{5}$$

where $a$ and $b$ are non zero integers.

Using (4) and (5) in (3) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2. \tag{6}$$

Equating real and imaginary parts, we have

$$u = 6a^2 - 42b^2 - 14ab,$$

$$v = a^2 - 7b^2 + 12ab.$$

Substituting the above values of $u$ and $v$ in (2), the values of $x$ and $y$ are given by

$$x = x(a, b) = 7a^2 - 49b^2 - 2ab,$$

$$y = y(a, b) = 5a^2 - 35b^2 - 26ab. \tag{7}$$

Thus (5) and (7) represent non zero distinct integral solutions of (1) in two parameters.

**Properties.**

- $y(a, 2a-1) - 5z(a, 2a-1) + 280t_{4,a} + 26t_{6,a} \equiv -70 \pmod{280}.$
- $x(a, a+1) - 7z(a, a+1) + 98t_{4,a} + 4t_{3,a} \equiv -98 \pmod{196}.$
- $y(a, 5a-3) + 5z(a, 5a-3) - 10t_{4,a} + 52t_{7,a} = 0.$
- $x(a, 3a-2) - y(a, 3a-2) - 2t_{4,a} + 126t_{4,a} - 24t_{8,a} \equiv -56 \pmod{168}.$
- $x(a, a+1) + 7z(a, a+1) - 14t_{4,a} + 2pr_a = 0.$
- $$x(a, 7a-5) + y(a, 7a-5) - 12t_{4,a} + 4116t_{4,a} + 56t_{9,a}$$
  $\equiv -2100((\mathrm{mod}\ 5880).$
- $y(a, 4a-3) - z(a, 4a-3) - 4t_{4,a} + 672t_{4,a} + 26t_{10,a} \equiv -378 \pmod{1008}.$
- $y(a^2, a+1) + z(a^2, a+1) - 6t_{4,a^2} + 28t_{4,a} + 52P_a^5 \equiv -28 \pmod{56}.$
- $z(a, 2a^2-1) + x(a, 2a^2-1) - 8t_{4,a} + 168t_{4,a^2} - 168t_{4,a} + 2SO_n \equiv -42.$
- $$x(a, a+1) + y(a, a+1) + z(a, a+1) - 13t_{4,a} + 77t_{4,a} + 28pr_a$$
  $\equiv -77 \pmod{154}.$

**Pattern 2**

Write 43 as

$$43 = (-6 + i\sqrt{7})(-6 - i\sqrt{7}). \tag{8}$$

Using (8) and (5) in (3) and employing the method of factorizations, define

$$(u + i\sqrt{7}v) = (-6 + i\sqrt{7})(a + i\sqrt{7}b)^2. \tag{9}$$

Equating real and imaginary parts, we have

$$u = -6a^2 + 42b^2 - 14ab,$$

$$v = a^2 - 7b^2 - 12ab.$$

Substituting the values of $u$ and $v$ in (2), the values of $x$ and $y$ are given by

$$x = x(a, b) = -5a^2 + 35b^2 - 26ab,$$

$$y = y(a, b) = -7a^2 + 49b^2 - 2ab. \tag{10}$$

Thus (5) and (10) represent non zero distinct integral solutions of (1) in two parameters.

**Pattern 3**

Consider (3) as

$$u^2 - 36z^2 = 7(z^2 - v^2). \tag{11}$$

Write (11) in the form of ratio as

$$\frac{u + 6z}{z + v} = \frac{7(z - v)}{u - 6z}$$

$$= \frac{\alpha}{\beta}, \quad \beta \neq 0$$

which is equivalent to the following two equations

$$u\beta - \alpha v + z(6\beta - \alpha) = 0,$$

$$-\alpha u - 7v\beta + z(6\alpha + 7\beta) = 0.$$

On employing the method of cross multiplication, we get

$$u = -6\alpha^2 + 42\beta^2 - 14\alpha\beta,$$

$$v = \alpha^2 - 12\alpha\beta - 7\beta^2, \quad (12)$$

$$z = -7\beta^2 - \alpha^2. \quad (13)$$

Substituting the values of $u$ and $v$ in (2), the non zero distinct integral values of $x$ and $y$ are given by

$$x = x(\alpha, \beta) = -5\alpha^2 + 35\beta^2 - 26\alpha\beta,$$

$$y = y(\alpha, \beta) = -7\alpha^2 + 49\beta^2 - 2\alpha\beta. \quad (14)$$

Thus (13) and (14) represent the nonzero distinct integer solutions of (1) in two parameters.

**Properties.**

- $x(a, a+1) - 5z(a, a+1) - 70t_{4,a} + 52t_{3,a} \equiv 70 \pmod{140}$.
- $x(a, a-1) + 5z(a, 3a-1) + 6t_{4,a} - 252t_{4,a} + 52t_{5,a} \equiv 28 \pmod{168}$.
- $y(a, 2a-1) - z(a, 2a-1) + 6t_{4,a} - 224t_{4,a} + 2t_{6,a} \equiv 56 \pmod{224}$.
- $y(a, 5a-3) + z(a, 5a-3) + 8t_{4,a} - 1050t_{3,a} + 4t_{7,a} \equiv 378 \pmod{1260}$.
- $x(a, 3a-2) - y(a, 3a-2) - 2t_{4,a} + 126t_{4,a} + 24t_{8,a} \equiv -56 \pmod{168}$.
- $x(a, 4a-3) + y(a, 4a-3) + 12t_{4,a} - 1344t_{4,a} + 28t_{10,a} \equiv 756 \pmod{2016}$.
- $y(a^2, a+1) + 7z(a^2, a+1) + 14t_{4,a^2} + 4p_a^5 \equiv 0$.
- $y(a, a+1) - 7z(a, a+1) - 98t_{4,a} + 2pr_a \equiv 98 \pmod{196}$.
- $x(a, 2a^2-1) - 13y(a, 2a^2-1) - 86t_{4,a} + 2408t_{4,a^2} - 2408t_{4,a} = -602$.
- $$x(a, 2a^2+1) + 13y(a, 2a^2+1) + 96t_{4,a} - 2688t_{4,a^2} - 2688t_{4,a} + 156oH_a$$
  $= 672$.

**Pattern 4**

Write (11) in the form of ratio as

$$\frac{u+6z}{7(z+v)} = \frac{(z-v)}{u-6z}$$

$$= \frac{\alpha}{\beta}, \quad \beta \neq 0$$

which is equivalent to the following two equations:

$$u\beta - \alpha 7v + z(6\beta - 7\alpha) = 0,$$

$$-\alpha u - \beta v + z(6\alpha + \beta) = 0.$$

On employing the method of cross multiplication, we get

$$u = -42\alpha^2 - 14\alpha\beta + 6\beta^2,$$

$$v = 7\alpha^2 - 12\alpha\beta - \beta^2, \tag{15}$$

$$z = -7\alpha^2 - \beta^2. \tag{16}$$

Substituting the values of $u$ and $v$ in (2), the non zero distinct integral values of $x$ and $y$ are given by

$$x = x(\alpha, \beta) = -35\alpha^2 - 26\alpha\beta + 5\beta^2,$$

$$y = y(\alpha, \beta) = -49\alpha^2 - 2\alpha\beta + 7\beta^2. \tag{17}$$

Thus (16) and (17) represent the nonzero distinct integer solutions of (1) in two parameters.

**Pattern 5**

Write (3) as

$$7v^2 = 43z^2 - u^2. \tag{18}$$

Assume

$$v = 43a^2 - b^2. \tag{19}$$

Write 7 as

$$7 = (\sqrt{43} + 6)(\sqrt{43} - 6). \tag{20}$$

Using (19) and (20) in (18), employing the method of factorization, define

$$(\sqrt{43}z + u) = (\sqrt{43} + 6)(\sqrt{43}a + b)^2. \tag{21}$$

Equating rational and irrational parts, we get

$$u = 258a^2 + 6b^2 + 86ab,$$

$$z = z(a, b) = 43a^2 + b^2 + 12ab. \tag{22}$$

Substituting the values of $u$ and $v$ in (2), we get

$$x = x(a, b) = 301a^2 + 5b^2 + 86ab,$$

$$y = y(a, b) = 215a^2 + 7b^2 + 86ab. \tag{23}$$

Thus (22) and (23) represent the integer solutions of (1).

**Properties:**

- $x(a, a+1) - y(a, a+1) - 86t_{4,a} + 2t_{4,a} \equiv -2 \pmod{4}$.
- $x(a, a+1) + y(a, a+1) - 516t_{4,a} - 12t_{4,a} - 344t_{3,a} \equiv 12 \pmod{24}$.
- $x(a, a+1) - 5z(a, a+1) - 86t_{4,a} + 60pr_a = 0$.
- $x(a, a+1) + 5z(a, a+1) - 516t_{4,a} - 10t_{4,a} - 292t_{3,a} \equiv 10 \pmod{20}$.
- $y(a, 3a-1) - 7z(a, 3a-1) + 86t_{4,a} - 4t_{5,a} = 0$.
- $y(a, 5a-3) + 7z(a, 5a-3) - 516t_{4,a} - 350t_{4,a} - 340t_{7,a} \equiv 126 \pmod{420}$.
- $y(a, 3a-2) + z(a, 3a-2) - 258t_{4,a} - 72t_{4,a} - 98t_{8,a} \equiv 32 \pmod{96}$.
- $y(a, 4a-3) - z(a, 4a-3) - 172t_{4,a} - 96t_{4,a} - 74t_{10,a} \equiv 54 \pmod{144}$.
- $z(a^2, a+1) + x(a^2, a+1) - 344t_{4,a^2} - 6t_{4,a} - 196p_a^5 \equiv 6 \pmod{12}$.

- $$x(a,\, 2a^2-1)+y(a,\, 2a^2-1)+z(a,\, 2a^2-1)$$

$$-559t_{4,a^2}-52t_{4,a^2}+52t_{4,a}-184so_a$$

$$=13.$$

**Pattern 6**

Consider (3) as

$$u^2+7v^2=43z^2\times 1. \tag{24}$$

Write 43 as

$$43=(6+i\sqrt{7})(6-i\sqrt{7}). \tag{25}$$

Write 1 as

$$1=\frac{(3+i\sqrt{7})(3-i\sqrt{7})}{16}. \tag{26}$$

Using (5), (25), (26) in (24) and employing the method of factorization, define

$$(u+i\sqrt{7}v)=(6+i\sqrt{7})(a+i\sqrt{7}b)^2\left(\frac{3+i\sqrt{7}}{4}\right).$$

Equating real and imaginary parts, we have

$$u=\frac{1}{4}[11a^2-77b^2-126ab],$$

$$v=\frac{1}{4}[9a^2-63b^2+22ab],$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $4a$, $b$ by $4b$, we have

$$z=4(a^2+7b^2),$$

$$u=11a^2-77b^2-126ab,$$

$$v=9a^2-63b^2+22ab. \tag{27}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = 20a^2 - 140b^2 - 104ab,$$

$$y = y(a, b) = 2a^2 - 14b^2 - 148ab. \tag{28}$$

Thus (27) and (28) represent the non zero distinct integral solutions of (1) in two parameters.

**Properties:**

- $x(a, a+1) - 10y(a, a+1) - 2752t_{3,a} = 0.$
- $2y(a, 3a-1) - z(a, 3a-1) + 504t_{4,a} + 592t_{5,a} \equiv -56 \pmod{336}.$
- $2y(a, 2a-1) + z(a, 2a-1) - 8t_{4,a} + 296t_{6,a} = 0.$
- $x(a, a+1) + y(a, a+1) - 22t_{4,a} + 154t_{4,a} + 252pr_a \equiv -154 \pmod{308}.$
- $x(a, 2a^2-1) + z(a, 2a^2-1) - 24t_{4,a} + 448t_{4,a^2} + 104so_a - 448t_{4,a} = -112.$
- $y(a, 2a^2+1) + z(a, 2a^2+1) - 6t_{4,a^2} - 56t_{4,a^2} - 56t_{4,a} + 444OH_a = 14.$
- $x(a, 5a-3) - y(a, 5a-3) - 18t_{4,a} + 3150t_{4,a} - 88t_{7,a} \equiv -1134 \pmod{3780}.$
- $x(a, 4a-3) - z(a, 4a-3) - 16t_{4,a} + 2688t_{4,a} + 104t_{10,a} \equiv -1512 \pmod{4032}.$
- $y(a, 7a-5) - z(a, 7a-5) + 2t_{4,a} + 2058t_{4,a} + 296t_{9,a} \equiv -1050 \pmod{2940}.$
- $x(a, 3a-2) + y(a, 3a-2) + z(a, 3a-2) - 26t_{4,a} + 1134t_{4,a} + 252t_{8,a}$

  $\equiv -504 \pmod{1512}.$

**Pattern 7**

Consider 1 as

$$1 = \frac{(3 + i4\sqrt{7})(3 - i4\sqrt{7})}{121}. \tag{29}$$

Using (25), (29), (5) in (24) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2\left(\frac{3 + i4\sqrt{7}}{11}\right).$$

Equating real and imaginary parts, we have

$$u = \frac{1}{11}[-10a^2 + 70b^2 - 378ab],$$

$$v = \frac{1}{11}[27a^2 - 189b^2 - 20ab].$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $11a$, $b$ by $11b$, we have

$$z = 121(a^2 + 7b^2),$$

$$u = -110a^2 + 770b^2 - 4158ab,$$

$$v = 297a^2 - 2079b^2 - 220ab. \tag{30}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = 187a^2 - 1309b^2 - 4378ab,$$

$$y = y(a, b) = -407a^2 + 2849b^2 - 3938ab. \tag{31}$$

Thus (30) and (31) represent the non zero distinct integral solutions of (1) in two parameters.

**Pattern 8**

Also, 1 is represented as

$$1 = \frac{(1 + i3\sqrt{7})(1 - i3\sqrt{7})}{64}. \tag{32}$$

Using (25), (32), (5) in (24) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2\left(\frac{1 + i3\sqrt{7}}{8}\right).$$

Equating real sand imaginary parts, we have

$$u = \frac{1}{8}[-15a^2 + 105b^2 - 266ab],$$

$$v = \frac{1}{8}[-30a^2 + 19b^2 - 133ab].$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $8a$, $b$ by $8b$, we have

$$z = 64(a^2 + 7b^2),$$

$$u = -120a^2 + 840b^2 - 2128ab,$$

$$v = 152a^2 - 1064b^2 - 240ab. \tag{33}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = 32a^2 - 224b^2 - 2368ab,$$

$$y = y(a, b) = -272a^2 + 1904b^2 - 1888ab. \tag{34}$$

Thus (33) and (34) represent the non zero distinct integral solutions of (1) in two parameters.

**Pattern 9**

Also, 1 is represented as

$$1 = \frac{(-3 + i\sqrt{7})(-3 - i\sqrt{7})}{16}. \tag{35}$$

Using (25), (5) and (35) in (24) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2\left(\frac{-3 + i\sqrt{7}}{4}\right).$$

Equating real and imaginary parts, we have

$$u = \frac{1}{4}[-25a^2 + 175b^2 - 42ab],$$

$$v = \frac{1}{4}[3a^2 - 21b^2 - 50ab].$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $2a$, $b$ by $2b$, we have

$$z = 4(a^2 + 7b^2),$$

$$u = -25a^2 + 175b^2 - 42ab,$$

$$v = 3a^2 - 21b^2 - 50ab. \tag{36}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = -22a^2 + 154b^2 - 92ab,$$

$$y = y(a, b) = -28a^2 + 196b^2 + 8ab. \tag{37}$$

Thus (36) and (37) represent the integer solutions of (1) in two parameters.

**Pattern 10**

Consider 1 as

$$1 = \frac{(-3 + i4\sqrt{7})(-3 - i4\sqrt{7})}{121}. \tag{38}$$

Using (5), (25) and (38) in (24) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2\left(\frac{-3 + i4\sqrt{7}}{11}\right).$$

Equating real and imaginary parts, we have

$$u = \frac{1}{11}[-46a^2 + 322b^2 - 294ab],$$

$$v = \frac{1}{11}[21a^2 - 147b^2 - 92ab].$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $2a$, $b$ by $2b$, we have

$$z = 121(a^2 + 7b^2),$$

$$u = -506a^2 + 3542b^2 - 3234ab,$$

$$v = 231a^2 - 1617b^2 - 1012ab. \tag{39}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = -275a^2 + 1925b^2 - 4246ab,$$

$$y = y(a, b) = -737a^2 + 5159b^2 - 2222ab. \tag{40}$$

Thus (39) and (40) represent the integer solutions of (1) in two parameters.

**Pattern 11**

Write 1 as

$$1 = \frac{(-1 + i3\sqrt{7})(-1 - i3\sqrt{7})}{64}. \tag{41}$$

Using (5), (25) and (41) in (24) and employing the method of factorization, define

$$(u + i\sqrt{7}v) = (6 + i\sqrt{7})(a + i\sqrt{7}b)^2\left(\frac{-1 + i3\sqrt{7}}{8}\right).$$

Equating real and imaginary parts, we have

$$u = \frac{1}{8}[-27a^2 + 189b^2 - 238ab],$$

$$v = \frac{1}{8}[17a^2 - 119b^2 - 54ab].$$

As our interest is on finding integer solutions, choose $a$ and $b$ so that $u$ and $v$ are integers. Replacing $a$ by $8a$, $b$ by $8b$, we have

$$z = 64(a^2 + 7b^2),$$

$$u = -216a^2 + 1512b^2 - 1904ab,$$

$$v = 136a^2 - 952b^2 - 432ab. \tag{42}$$

Substituting $u$ and $v$ in (2), we have

$$x = x(a, b) = -80a^2 + 560b^2 - 2336ab,$$

$$y = y(a, b) = -352a^2 + 2464b^2 - 1472ab. \tag{43}$$

Thus (42) and (43) represent the integer solutions of (1) in two parameters.

## 3. Conclusion

In this paper, we have presented different patterns of integer solutions to the ternary quadratic equation $2(x^2 + y^2) - 3xy = 43z^2$ representing the cone. As the Diophantine equations are rich in variety, one may attempt to find integer solutions to other choices of equations along with suitable properties.

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