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ON THE ADJACENT VERTEX-DISTINGUISHING EDGE COLORING OF $C_m \cdot F_n$

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Abstract

Supposing $C_m = u_1 u_2 \cdots u_n v_1$,

$$V(C_m \cdot F_n) = \{u_i \mid i = 1, 2, \ldots, m\} \cup \{v_{ij} \mid i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n\},$$

$$E(C_m \cdot F_n) = E(C_m) \cup \{u_i v_{ij} \mid i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n\} \cup \{v_{ij} v_{i(j+1)} \mid i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n-1\}.$$

In this paper, we present Adjacent Vertex-distinguishing Edge Chromatic Number of $C_m \cdot F_n$ $(n \geq 2)$.

1. Introduction

We discussed adjacent vertex-distinguishing edge coloring of graph in [1]-[3], it is very difficult question. We introduce the concept of adjacent vertex-distinguishing edge coloring, though it is easier than vertex-distinguishing edge coloring, it is very difficult too.

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Definition 1 [1]. G is a simple graph and k is a positive integer, if it exists a mapping f, $E(G) \xrightarrow{f} \{1, 2, \ldots, k\}$, and satisfied with $f(e) \neq f(e')$ for adjacent edge $e, e' \in E(G)$, then f is called a *Proper Edge Coloring* of G, is abbreviated k-PEC of G, and

$$\chi'(G) = \min\{k \mid k\text{-PEC of } G\}$$

is called the *Edge Chromatic Number* of G.

Definition 2 [2-5]. For the proper edge coloring f of simple graph, if it is satisfied with $C(u) \neq C(v)$ for $V(G)(u \neq v)$, where $C(u) = \{f(uv) \mid uv \in E(G)\}$, then f is called the *Vertex-distinguishing Edge Coloring*, is abbreviated k-VDEC of G, and

$$\chi'_{vd}(G) = \min\{k \mid k\text{-VDEC of } G\}$$

is called the *Vertex-distinguishing Edge Chromatic Number* of G.

Conjecture. G is a connected graph where $|V(G)| \geq 3$, if $G \neq C_5$ (5-cycle), then

$$\chi'_{as} \leq \Delta(G) + 2.$$

In which $\Delta(G)$ is maximum degree of G.

Definition 3. For a graph G, n_i is the vertex number which degree is i, using δ, Δ denoted the minimum, maximum degree of G, it is called

$$\mu(G) = \max\{ \min\{\lambda \mid \binom{\lambda}{i} \geq n_i\}, \sigma \leq i \leq \Delta\},$$

Combinatorial Degree of G.

Conjecture. For connected graph G and $|V(G)| \geq 3$, then

$$\mu(G) \leq \chi'_{vd}(G)$$

$$\leq \mu(G) + 1$$

the left of the conjecture is obviously true.

Definition 4 [6]**.** Supposing G and H are two simple graphs which are vertex disjointed and edge disjointed,

$$V(G \vee H) = V(G) \cup V(H),$$

$$E(G \vee H) = E(G) \cup E(H) \cup \{uv \mid u \in V(G),\ v \in V(H)\},$$

then $G \vee H$ is called *Join-graph* of G and H.

In this paper, we discuss Adjacent Vertex-distinguishing Edge Chromatic Number of $C_m \cdot F_n$. The terms and signs we use in this paper but not denoted can be found in [5] and [6].

2. Main Results

Lemma 1 [4]**.** *G is a connected graph where $|V(G)| \geq 3$, if there are vertices of maximum degree which is adjacent, then*

$$\chi'_{as} \geq \Delta(G) + 1.$$

Theorem 1. *If* $n \geq 2$, $\chi'_{as}(C_m \cdot F_n) = n + 3$.

Proof. Clearly, $\Delta(C_m \cdot F_n) = n + 2$, we obtain $\chi'_{as}(C_m \cdot F_n) \geq n + 3$ by Lemma 1. In order to prove the result is true, we need prove $C_m \cdot F_n$ is $(n + 3)$-AVDEC only.

Case 1. If $m = 0(\text{mod}\,3)$. Suppose *f* is

$$u_1u_2,\ u_2u_3,\ \ldots,\ u_nu_1,$$

we can color the edges with colors $1, 2, 3,$ repeatedly.

Case 1.1. If $n = 2$.

For $i \equiv 1(\text{mod}\,3)$,

$$f(u_iv_{i1}) = 2,$$

$$f(v_iv_{i2}) = 4,$$

$$f(v_{i1}v_{i2}) = 1.$$

For $i \equiv 2 (\bmod 3)$,

$$f(u_i v_{i1}) = 3,$$

$$f(u_i v_{i2}) = 5,$$

$$f(v_{i1} v_{i2}) = 2.$$

For $i \equiv 0 (\bmod 3)$,

$$f(u_i v_{i1}) = 4,$$

$$f(u_i v_{i2}) = 5,$$

$$f(v_{i1} v_{i2}) = 3.$$

For f, we have

$$C(u_i) \neq C(v_{ij}) \quad (i = 1, 2, \ldots, m;\ j = 1, 2),$$

$$C(v_{i1}) \neq C(v_{i2}) \quad (i = 1, 2, \ldots, m).$$

Suppose $\overline{C}(u_i) = \{1, 2, 3, 4, 5\} \backslash C(u_i)$. Then

$$\overline{C}(u_i) = \{5\}, \quad i \equiv 1 (\bmod 3);$$

$$\overline{C}(u_i) = \{4\}, \quad i \equiv 2 (\bmod 3);$$

$$\overline{C}(u_i) = \{1\}, \quad i \equiv 0 (\bmod 3).$$

So f is a mapping about 5-AVDEC of $C_m \cdot F_2$, this proves that the result is true.

Case 1.2. If $n = 3$.

For $i = 1 (\bmod 3)$,

$$f(u_i v_{i1}) = 2,$$

$$f(u_i v_{i2}) = 4,$$

$$f(u_i v_{i3}) = 5,$$

$$f(v_{i1} v_{i2}) = 1,$$

$$f(v_{i2} v_{i3}) = 3.$$

For $i \equiv 2 (\bmod 3)$,

$$f(u_i v_{i1}) = 3,$$

$$f(u_i v_{i2}) = 5,$$

$$f(u_i v_{i3}) = 6,$$

$$f(v_{i1} v_{i2}) = 1,$$

$$f(v_{i2} v_{i3}) = 2.$$

For $i \equiv 0 (\bmod 3)$,

$$f(u_i v_{i1}) = 4,$$

$$f(u_i v_{i2}) = 5,$$

$$f(u_i v_{i3}) = 6,$$

$$f(v_{i1} v_{i2}) = 2,$$

$$f(v_{i2} v_{i3}) = 3.$$

For f, same as Case 1.1, we need check $C(u_i) \neq C(u_{i+1})$ $(i = 1, 2, \ldots, m-1)$ and $C(u_1) \neq C(u_m)$ only. Then

$$\overline{C}(u_i) = \{6\}, \quad i \equiv 1 (\bmod 3);$$

$$\overline{C}(u_i) = \{4\}, \quad i \equiv 2 (\bmod 3);$$

$$\overline{C}(u_i) = \{1\}, \quad i \equiv 0 (\bmod 3).$$

Hence f is a mapping about 6-AVDEC of $C_m \cdot F_3$, this proves that the result is true.

Case 1.3. If $n \geq 3$.

For $i \equiv 1 (\bmod 3)$,

$$f(u_i v_{i1}) = 2, \quad f(u_i v_{ij}) = j + 2 \quad (j = 2, 3, \ldots, n);$$

$$f(v_{i1} v_{i2}) = 1, \quad f(v_{ij} v_{i(j+1)}) = j + 1 \quad (j = 2, 3, \ldots, n-1).$$

For $i \equiv 2(\bmod 3)$,

$$f(u_i v_{ij}) = j + 3 \quad (j = 1, 2, \ldots, n);$$

$$f(v_{ij} v_{i(j+1)}) = j + 1 \quad (j = 1, 2, \ldots, n - 1).$$

For f, same as Case 1.1, we need check $C(u_i) \neq C(u_{i+1})$ $(i = 1, 2, \ldots, m - 1)$ and $C(u_1) \neq C(u_n)$ only. Then

$$\overline{C}(u_i) = \{n + 3\}, \quad i \equiv 1(\bmod 3);$$

$$\overline{C}(u_i) = \{4\}, \quad i \equiv 2(\bmod 3);$$

$$\overline{C}(u_i) = \{1\}, \quad i \equiv 0(\bmod 3).$$

Hence f is a mapping about $(n + 3)$-AVDEC of $C_m \cdot F_n$ $(n \geq 3)$, this proves that the result is true.

Case 2. If $m \equiv 1(\bmod 3)$, suppose f is

$$u_1u_2, u_2u_3, \ldots, u_mu_1.$$

First we can color the edges with colors 1, 2, 3, 4, then color the edges with colors 1, 2, 3, repeatedly.

For $u_i v_{ij}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ and $v_{ij} v_{i(j+1)}$ $(i = 1, 2, \ldots, m;$ $j = 1, 2, \ldots, n - 1)$, we can color the edges like Case 1, we can obtain $C_m \cdot F_n$ is $(n + 3)$-AVDEC. This proves that the result is true.

Case 3. If $m \equiv 2(\bmod 3)$, suppose f is

$$u_1u_2, u_2u_3, \ldots, u_nu_1.$$

First we can color the edges with colors 1, 2, 3, 4, 5, then color the edges with colors 1, 2, 3, repeatedly.

The rest edges can be colored like Case 1, we can obtain $C_m \cdot F_n$ is $(n + 3)$-AVDEC. This proves that the result is true.

All in all, the theorem is true.

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