JP Journal of Mathematical Sciences
Volume 35, Issue 2, 2024, Pages 37-56

THE CAUCHY PROBLEM FOR THE MINIMAL SURFACE EQUATION

Boureima Bella, Bruno Kabore and Ibrahim Ly

Abstract

By means of a variational approach, we discuss the Cauchy problem related to the minimal surface operator which is a nonlinear elliptic operator. To cite a few examples, this operator appears in physics, differential geometry, fluid mechanics, flows in porous media, and in many other areas. The Cauchy data are given on a part $\mathcal{S}$ of the boundary surface which characterizes the Ill-posedness (cf. the manuscripts of Hadamard and Tarkhanov) of the problem. In the case of real analytic data on a non-characteristic hypersurface, the existence of a local solution to the Cauchy problem near $\mathcal{S}$ is guaranteed by the Cauchy-Kowalewskaya theorem. To each variational problem, there exists a possibility to assign to it an estimate called a Korn estimate. Therefore, to study our problem, we develop a Korn estimate which is a crucial tool for the existence of a minimizer of the functional we will be considering. We prove necessary and sufficient conditions for the solvability of our Cauchy problem. Moreover, we invoke purely nonlinear methods such as successive iterations in the Sobolev space $W^{1,2}$ which is a reflexive Banach space to show how to construct an approximate solution which is the last result of our paper.

Keywords and phrases: nonlinear PDE, Cauchy problem, variational problems, elliptic operators.

Received April 4, 2024; Accepted May 3, 2024

References

[1] J. Cringanu, The p-Laplacian on Sobolev spaces $W^{1,p}(\Omega)$, Studia Univ. "Babeş-Bolyai", Mathematical 1 (2005), 25-31, https://www.researchgate.net/publication/268860803.

[2] B. Dacorogna, Introduction to the Calculus of Variations, Imperial College Press, London, 2004, DOI:10.1142/p361.

[3] J. I. Diaz, Nonlinear partial differential equations and free boundaries, Vol. I, Elliptic Equations Research Notes in Mathematics, Vol. 106, Pitman, Boston, MA, 1985, https://doi.org/10.1002/zamm.19870670311.

[4] Lawrence C. Evans, Partial differential equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, Providence, RI, 1998, www.ams.org/bookpages/gsm-19.

[5] J. Hadamard, Quelques cas d'impossibilité du problème de Cauchy, Memorial N. I. Lobachevsky 2 (1927), 163-176, http://www.mathnet.ru/rus/agreement.

[6] V. A. Kozlov, V. G. Maz'ya and A. V. Fomin, An iterative method for solving the Cauchy problem for elliptic quations, Comput. Maths. Math. Phys. 31(1) (1991), 45-52, https://www.researchgate.net/publication/234811978.

[7] A. V. Kryanev, An iterative method for solving ill-posed problems, Comput. Maths. Math. Phys. 14(1) (1974), 25-35, https://doi.org/10.1016/0041-5553(74)90133-5.

[8] A. L. Krylov, The Cauchy problem for Laplace's equation in the complex domain, Soviet Math. Dokl. 10 (1969), 1184-1187, http://www.mathnet.ru/eng/agreement.

[9] P. Kügler and A. Leitao, Mean value iterations for nonlinear elliptic Cauchy problems, Numer. Math. 96 (2003), 269-293, https://doi.org/10.1007/s00211-003-0477-6.

[10] I. Ly and N. Tarkhanov, The Cauchy problem for nonlinear elliptic equations, Nonlinear Anal. 70(7) (2009), 2494-2505, url=https://api.semanticscholar.org/CorpusID:116973503.

[11] I. Ly, An iterative method for solving the Cauchy problem for the p-Laplace operator, Complex Variables and Elliptic equations 55(11) (2010), 1079-1088, https://doi.org/10.1080/17476931003628257.

[12] V. P. Maslov, The existence of a solution of an ill-posed problem is equivalent to the convergence of a regularization process, Uspekhi Mat. Nauk 23(3) (1968), 183-184, http://www.mathnet.ru/eng/agreement.

[13] Charles B. Morrey, Multiple Integrals in the Calculus of Variations, Springer-Verlag, Berlin, 1966, https:// DOI 10.1007/978-3-540-69952-1.

[14] C. Morrey, On the analyticity of the solutions of analytic nonlinear elliptic systems of partial differential equations. II. Analyticity at the boundary, Amer. J. Math. 80 (1958), 219-237, https://doi.org/10.2307/2372830.

[15] I. Shestakov, On the Zaremba problem for the p-Laplace operator, Contemporary Mathematics 591 (2013), http://dx.doi.org/10.1090/conm/591/11841.

[16] N. Tarkhanov, The Cauchy Problem for Solutions of Elliptic Equations, Akademie Verlag, Berlin, 1995, url=https://api.semanticscholar.org/CorpusID:116973503.

[17] M. Taylor, Partial differential operators III, Nonlineare Equations, Springer-Verlag, Berlin, 1996, http://doi.org/10.1007/978-1-4187-2.

[18] L. R. Volevich and A. R. Shirikyan, Stable and unstable manifolds for non-linear elliptic equations with parameter, Trans. Moscow Math. Soc. 61 (2000), 97-138, DOI: 10.1070/RM2004v059n05ABEH000789.

[19] K. Yosida, Functional Analysis, Springer-Verlag, Berlin and New York, 1965, https://doi.org/ 10.1007/978-3-662-11791-0.

[20] S. Zaremba, Sur un problème mixte relatif à l'équation de Laplace, Bull. Intern. Acad. Sci. Cracovie, Classe des Sciences Mathématiques et Naturelles, Série A (1910), 313-344, DOI: 10.4467/2353737XCT.15.217.4422.