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THE FINE $\mathfrak{spo}(2|n)$- EQUIVARIANT QUANTIZATIONS ON THE SUPER CIRCLES $S^{1|n}$

Aboubacar Nibirantiza

Abstract

In this paper, we generalize the known results on the super circles $S^{1|1}$ and $S^{1|2}$. We construct the fine equivariant quantization on the super circles $S^{1|n}$ for $n \geq 3$. The equivariant Lie superalgebra is $\mathfrak{spo}(2|n)$ which is constituted of the contact projective vector fields on $S^{1|n}$. In order to construct the fine equivariant quantization on $S^{1|n}$, we use the model developed, in the purely even case, by Conley and Ovsienko in [1]. We also use the technical of Casimir operators to prove the uniqueness of the fine quantization on $S^{1|n}$. The technical of Casimir operators used here is the same as the one used by Mathonet and Radoux in [12] to prove the existence of a $\mathfrak{pgl}(p+1|q)$- equivariant quantization on $\mathbb{R}^{p|q}$.

Keywords and phrases: contact structure on superspaces, Lie superalgebras, differential operators, equivariant quantization.

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