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THE RIEMANNIAN STRUCTURE OF THE THREE-PARAMETER LOGNORMAL DISTRIBUTION

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Abstract

In this paper, we will utilize the results already known in differential geometry and provide an intuitive understanding of the lognormal distribution. This approach leads to the definition of new concepts to provide new results of statistical importance. These results may be of particular interest to mathematical physicists. In general, it has been shown that the parameter space is not of constant curvature. We found that all three sectional, mean and scalar curvatures are a complicated function of the shape parameter of lognormal distribution. In addition, we calculated some invariant quantities, such as sectional curvature, Ricci curvature, mean curvature and scalar curvature.

Keywords and phrases: mean curvature, lognormal distribution, Ricci curvature, Riemannian geometry, scalar curvature, sectional curvature.

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